The answer is $\max[v_2(n), v_5(n)]$.

If $n = 2^\alpha 5^\beta$ for some non-negative integers α, β, then the result is clear. Now assume that $\gcd(n, 10) = 1$. First we understand what we are doing when we divide 1 by n. First we put in 0 since $n \cdot 0 < 1$ is the smallest mulitple of n less than 1. After we put the decimal point, the new dividend becomes 10, so we put the number n_1 in the quotient, where nn_1 is the largest multiple of n less than 10. Now the dividend is $(10 - nn_1) \cdot 10$, and the number to go in the quotient is nn_2, where n_2 is the largest multiple of n less than $(10 - nn_1) \cdot 10$, and we continue like this. Note that $n_1, n_2, \dots \in \{0, 1, \dots, 9\}$. We write this down as a sequence of steps as follows:

$$(10, n_1 n)$$

$$\mapsto (10^2 - 10n_1 n, n_2 n)$$

$$\mapsto (10^3 - 10^2 n_1 n - 10 n_2 n, n_3 n)$$

$$\vdots$$

Note that in every step, the difference, i.e., the first number of the pair, is as small as possible. Now choose x such that the repunit (numbers of the form $111\dots1$) with x digits is divisible by n. Call this repunit r_x. We can always do so as long as $\gcd(n, 10) = 1$. We let our sequence go on for x steps, and the last pair is

$$(10^x - n(10^{x-1}n_1 + 10^{x-2}n_2 + \cdots + 10n_{x-1}), n_x n).$$

The difference, i.e., the first number is positive and as small as possible. This means that $n(10^{x-1}n_1 + 10^{x-2}n_2 + \cdots + 10n_{x-1})$ is as large as possible. This number is also less than 10^x, which means it must have x digits. The largest number with x digits is $9r_x$. We show that $n(10^{x-1}n_1 + 10^{x-2}n_2 + \cdots + 10n_{x-1}) = 9r_x$. Note that $r_x < 10^x$, so $\frac{r_x}{n} < 10^{x-1}$, and $9\frac{r_x}{n} < 10 \cdot 10^{x-1} = 10^x$. So the number of digits of $9\frac{r_x}{n}$ is less than or equal to x. Since $10^{x-1}n_1 + 10^{x-2}n_2 + \cdots + 10n_{x-1}$ is an x digit number, the value of $n(10^{x-1}n_1 + 10^{x-2}n_2 + \cdots + 10n_{x-1})$ can be made $9r_x$, and thus we have $n(10^{x-1}n_1 + 10^{x-2}n_2 + \cdots + 10n_{x-1}) = 9r_x = 10^x - 1$ due to our as-large-as-possible criterion. So the last step is actually

$$(10^x - (10^x - 1), n_x n) = (1, 0 \cdot n) = (1, 0)$$

and thus the next step is

$$(10, n_1 n).$$

This means that the digits of the quotient again starts repeating from n_1, and as such the length of the non-repeating part is 0. If $n = 2^a 5^b k$ where $a, b \geq 0$ and $\gcd(k, 10) = 1$, then $\frac{1}{n} = \frac{1}{2^a 5^b} \cdot \frac{1}{k}$. Since the non-repeating part of $\frac{1}{k}$ is 0, the non-repeating part of $\frac{1}{n}$ is simply $\max(a, b)$.

Thus we have our answer as $\max[v_2(n), v_5(n)]$.