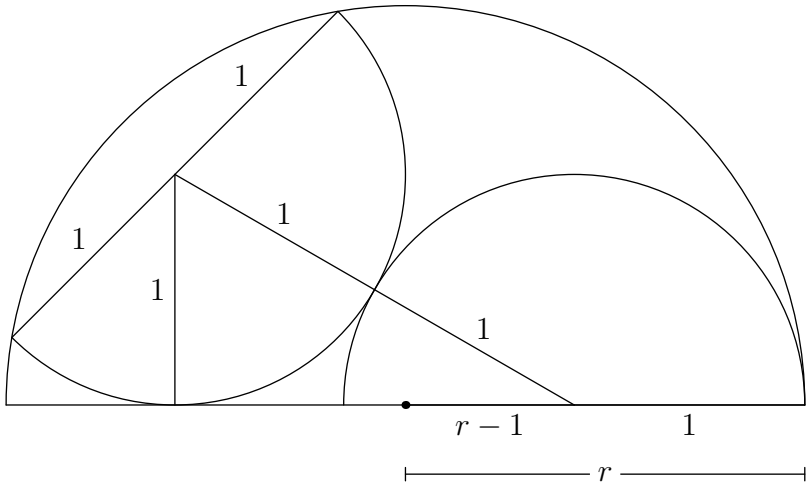
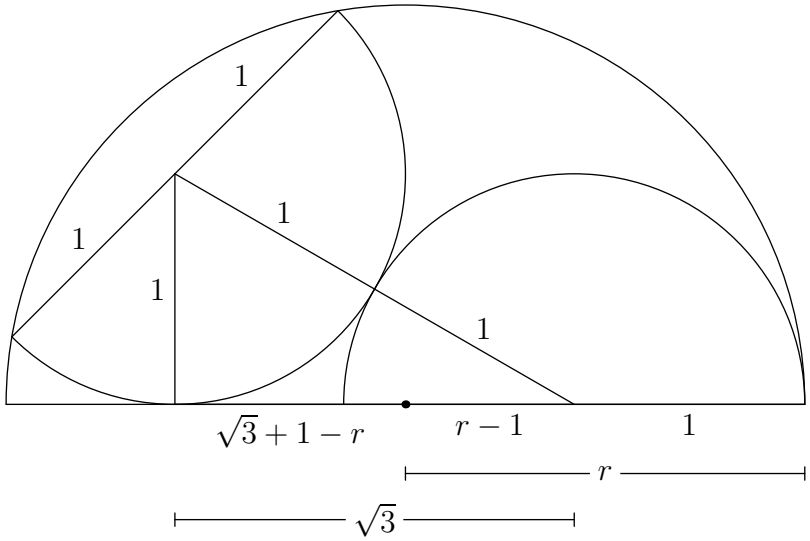
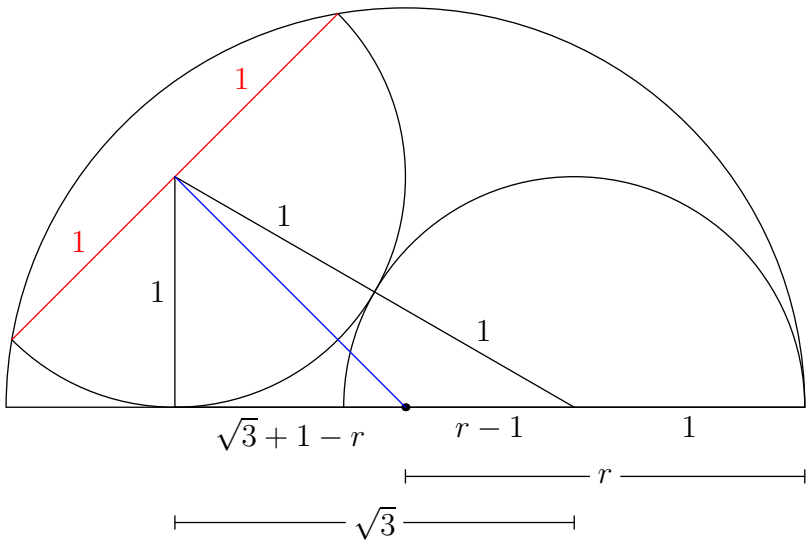
wlog, let the smaller radius be 1, and the larger radius be r. we just need to solve for r, after that finding the ratio of areas will be pretty simple. here are some lengths we can fill in immediately:



there's a 30-60-90 triangle just waiting to be used, so we can fill in two more lengths based on that:



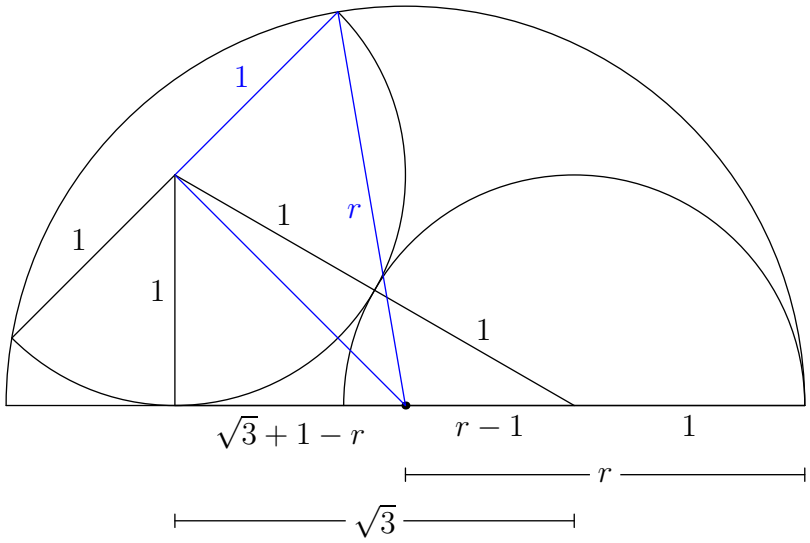
but there are still more right triangles waiting to be used! we just need to draw in more lines:



now, we know that

$$\text{blue}^2 = 1^2 + (\sqrt{3}+1-r)^2 = 5 + 2\sqrt{3} - 2r(\sqrt{3}+1) + r^2$$

it also seems like the blue line is perpendicular to the red line. this is true because the blue line bisects the red line, and a radius which bisects a chord must be perpendicular to that chord. so we might be able to use the pythagorean theorem again!



from the blue right triangle, we have

$$\left[5 + 2\sqrt{3} - 2r(\sqrt{3}+1) + r^2\right] + 1^2 = r^2$$

the r^2 terms cancel out, so we end up with a nice linear equation which can be solved to get $r = \sqrt{3}$. thus, the ratio of shaded area to total area is

$$\frac{\frac{1}{2}\pi(1)^2 + \frac{1}{2}\pi(1)^2}{\frac{1}{2}\pi(\sqrt{3})^2} = \frac{2}{3},$$

and we're done.