2019 AoPS Mock F=ma Summer Exam

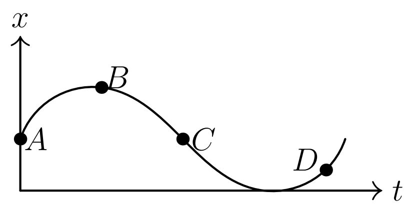
September 11-26

Instructions

- Use $g = 10$ N/kg
- Select the best answer choice for each question.
- Correct answers will be rewarded one point; incorrect answers and leaving your answer blank will be rewarded 0 points.
- A hand held caclulator may be used. It's memory must be cleared of any data or programs.
- No cellphones, textbooks or tables of formulas can be used in the exam.
- The problems on the test cannot be discussed or given to others before September 26 12:00 AM USA CT.
- The test should be taken in Olympiad conditions, or in other words you cannot leave the test until 75 minutes have passed.
- Don't cheat! This is a mock exam! Meaning you are taking it for extra practice to reach your goals!
- When done with the test please PM your answers (not solutions!) from each respective problem to me

Testwriters: Lol_man000, monkeycalculator, cmsgr8er and ojaswupadhyay

1. In the x vs. t plot shown below which of the following point(s) correspond to zero acceleration?



(A) B, D

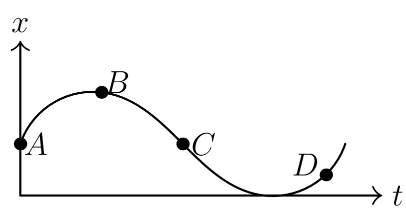
(B) C

(C) B

(D) A, B

(E) C, D

2. From the previous problem the graph was graphed by a student named YunYun. Her device for measuring position is very accurate, so that its reading contributes negligible uncertainty to her result. However, she is only able to activate the device to within an uncertainty in time Δt. Which of the following points has the least error as a measurement of time?



(A) A

(B) B

(C) C

(D) D

(E) A, C

3. A shell flying with velocity $v = 600$ m/s bursts into four identical fragments, so that the kinetic energy of the system increases by $\eta = \frac{4}{3}$ times. What maximum velocity can one of the fragments obtain?

(A) 1800 m/s

(B) 2000 m/s

(C) 2400 m/s

(D) 3000 m/s

(E) 4000 m/s

4. The drag force F_d on a sphere moving quickly through a non-viscous fluid depends on the density ρ, the radius R, and the speed v. Which of the following quantities is F_d proportional to?

(A) ρv

(B) $\rho R v$

(C) $\rho R v^2$

(D) $\rho R^2 v$

(E) $\rho R^2 v^2$

5. 20 children, each of weight 356 N, make a log raft by lashing together logs of diameter 0.30 m and length 1.80 m. What is the fewest number of logs needed to keep them afloat in the fresh water? Take the density of the logs to be 800 kg/m^3 and the density of fresh water to be 1000 kg/m^3.

(A) 25

(B) 26

(C) 27

(D) 28

(E) 29

6. A drain plug is opened in the bottom of a rolling cart allowing the accumulated sand in the cart to run out. What is true about the speed, momentum and kinetic energy of the rolling cart?

(A) The speed of the cart increases, while the momentum and the kinetic energy of the cart decreases

(B) The speed and the kinetic energy of the cart increases while the momentum decreases

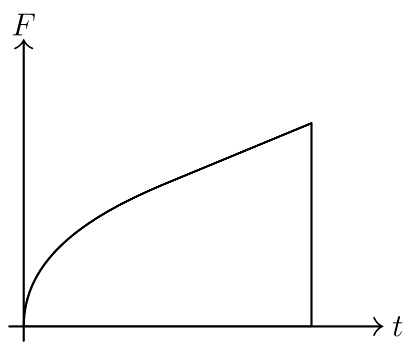
(C) The speed of the cart remains unchanged, while the momentum and kinetic energy of the cart decreases

(D) The momentum and the speed of the cart remains unchanged while the kinetic energy of the cart decreases

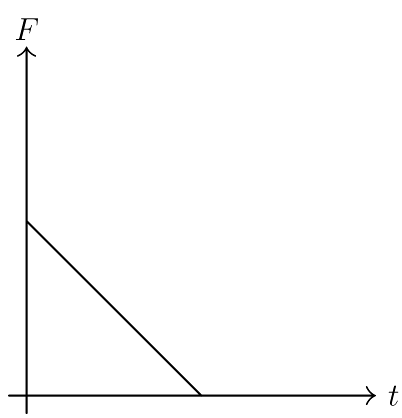
(E) All three quantities decrease with time

7. Which of the following Force vs. Time graphs represent the collision of a tennis ball with a wall?

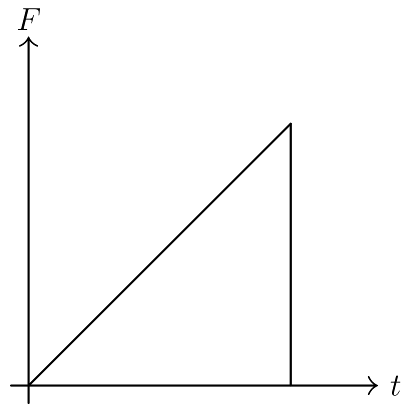
(A)



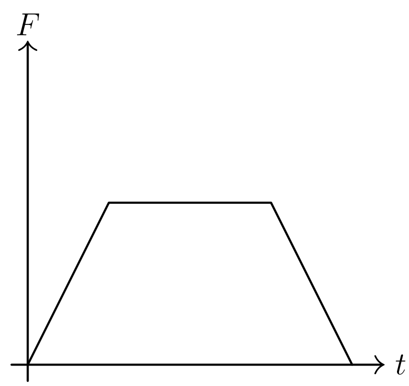
(B)



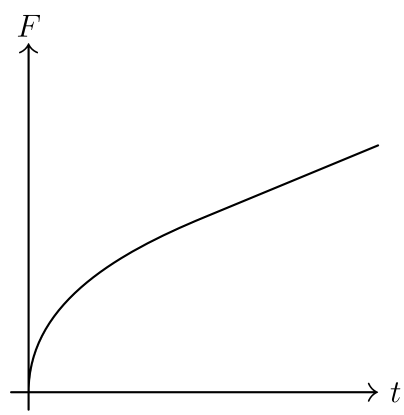
(C)



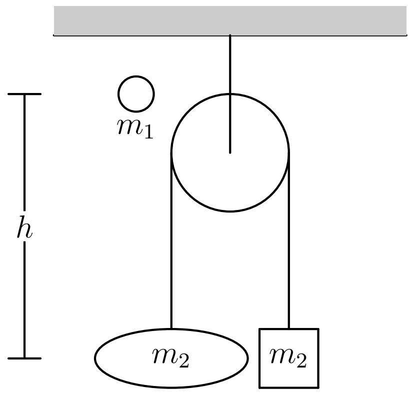
(D)



(E)



8. A ball of mass m_1 is dropped from a height h above a pan of mass m_2, where the ball is attached by a pulley to a block of mass m_2. The ball stick to the pan after collision. Find the final velocity of the pan.



(A) $\dfrac{m_2\sqrt{2gh}}{m_1+2m_2}$

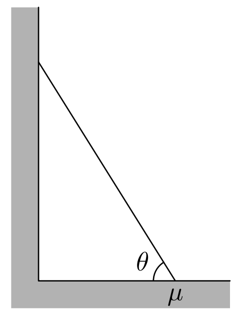
(B) $\dfrac{m_2\sqrt{4gh}}{m_1+2m_2}$

(C) $\dfrac{m_1\sqrt{4gh}}{m_1+2m_2}$

(D) $\dfrac{m_1\sqrt{2gh}}{m_2+3m_2}$

(E) $\dfrac{m_1\sqrt{2gh}}{m_1+2m_2}$

9. A ladder leans against a frictionless wall. If the coefficient of friction with the ground is μ, what is the smallest angle the ladder can make with the ground and not slip?



(A) $\tan^{-1}\left(\frac{1}{\mu}\right)$

(B) $\tan^{-1}\left(\frac{1}{2\mu}\right)$

(C) $\cos^{-1}\left(\frac{1}{\mu}\right)$

(D) $\cos^{-1}(\mu)$

(E) $\tan^{-1}\left(\frac{2}{3\mu}\right)$

10. Megumin decides that she wants to figure out g, the acceleration of gravity, by measuring the time t for the stone to fall down a height h above the ground. After making several timings, she concludes that

$$t = 1.6 \pm 0.1 \text{ s}$$

and she measures the height h as

$$h = 46.2 \pm 0.3 \text{ ft}$$

She then calculates g using $g = \frac{2h}{t^2} = 36.1$ fts^2 What is the absolute uncertainty in her answer?

(A) 0.6 ft/s^2

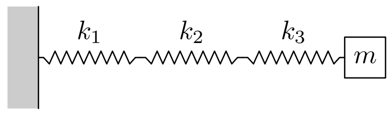
(B) 1.3 ft/s^2

(C) 1.7 ft/s^2

(D) 2.1 ft/s^2

(E) 4.8 ft/s^2

11. Three springs are joined and connected to a block of mass m as shown below. If the springs separately have spring constant k_1, k_2 and k_3 respectively, than what is the frequency of the oscillations of the block in terms of $f_1 = 0.5$ Hz, $f_2 = 2$ Hz and $f_3 = 3.2$ Hz if each frequency is defined as the frequency the block would have if connected to a only spring $1, 2$ or 3?



(A) 0.4 Hz

(B) 0.5 Hz

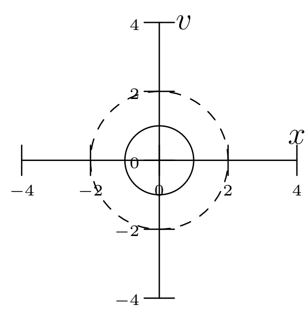
(C) 0.6 Hz

(D) 1.1 Hz

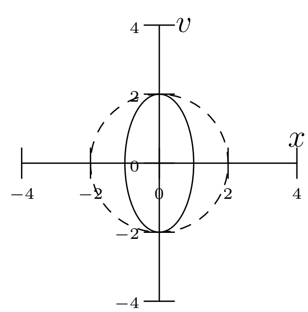
(E) 1.2 Hz

12. Suppose that a mass-spring system with mass m_0, spring constant k_0 is oscillating in simple harmonic motion. The position vs velocity of the system is plotted with the dotted line. Now, consider a new system with mass $m = 4m_0$ and $k = k_0$. Which of the following graphs can possibly represent the position vs velocity of the new system, in solid line?

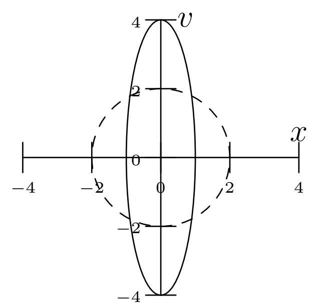
(A)



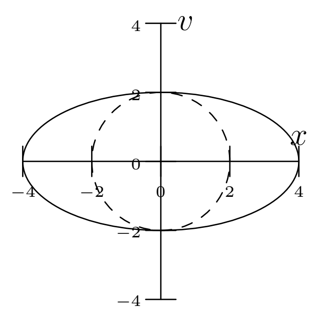
(B)



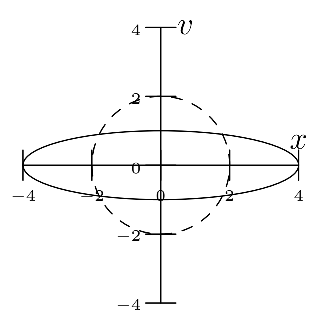
(C)



(D)



(E)



13. A liquid with a density half that of water is added to one side of a U-tube until the total length of the liquid is equal to the total length of water. Determine the equilibrium height difference between the two sides. This is given that the length of the total water column is L.

(A) L

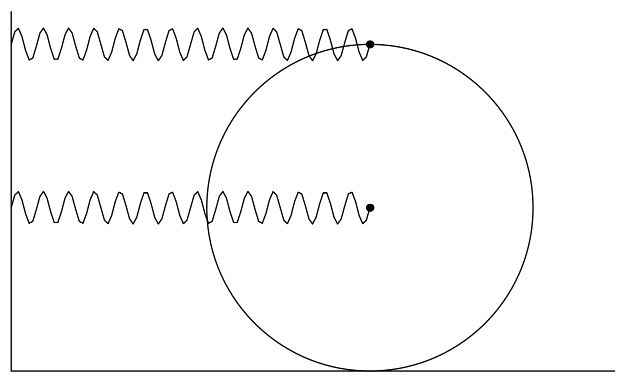
(B) $L/2$

(C) $L/4$

(D) $3L/4$

(E) $L/3$

14. Consider a circular cylinder of uniform density and total mass m, with two springs attached to the center and top. The springs have a spring constant k, and is attatched to a wall. Given that the cylinder rolls without slipping, find the oscillation period of the cylinder for small oscillations.



(A) $\pi\sqrt{\frac{6m}{5k}}$

(B) $\pi\sqrt{\frac{4m}{3k}}$

(C) $\pi\sqrt{\frac{3m}{2k}}$

(D) $\pi\sqrt{\frac{8m}{5k}}$

(E) $\pi\sqrt{\frac{2m}{k}}$

15. A spring with a spring constant k hangs vertically from the ceiling, initially at it's relaxed length. Kazuma attaches a mass m to the end and brings it down to a position that is $4mg/k$ below the initial position. He then lets go. What is the upward acceleration of the mass right after he lets it go?

(A) 0

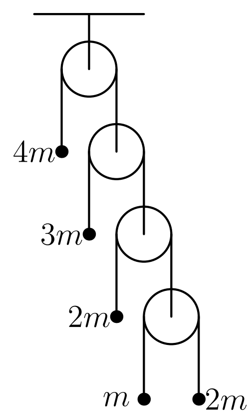
(B) g

(C) $2g$

(D) $3g$

(E) $4g$

16. Consider the quadruple Atwood's machine shown below. What is the acceleration of the top leftmost mass?



(A) $13g/15$

(B) $17g/15$

(C) $43g/149$

(D) $53g/67$

(E) $69g/274$

17. A working 1 : 10 scale model of a helicopter is lifted into the sky when it is powered by a 30 watt engine. What would be the minimum power needed to lift a life sized helicopter made from the same materials?

(A) 300 W

(B) 95 W

(C) 234 W

(D) 150 W

(E) 65 W

18. Let a satellite of mass m be in orbit around a planet of mass M. Let a be the semimajor axis of the orbit, what is the velocity of the satellite when it is a distance r away? (Express your answer in terms of r, M, and a)

(A) $GM\left(\frac{3}{r} - \frac{1}{a}\right)$

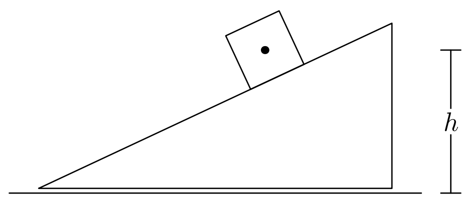
(B) $GM\left(\frac{1}{r} - \frac{2}{a}\right)$

(C) $GM\left(\frac{1}{r} - \frac{1}{a}\right)$

(D) $GM\left(\frac{2}{r} - \frac{1}{a}\right)$

(E) $GM\left(\frac{2}{r} - \frac{2}{a}\right)$

The following information applies to both 19 and 20: A block of mass $m = 0.5$ kg sits on top of a wedge of mass $M = 2$ kg at a height $h = 1.4$ m at an incline of $\theta = 30°$. Assume everything moves without friction.



19. What is the speed of the wedge, right, after the block is released?

(A) 1.17 m/s

(B) 3.38 m/s

(C) 2.79 m/s

(D) 5.67 m/s

(E) 4.32 m/s

20. What is the acceleration of the wedge, after the block is released?

(A) 0.67 m/s^2

(B) 1.21 m/s^2

(C) 1.43 m/s^2

(D) 0.32 m/s^2

(E) 0.94 m/s^2

21. Bob lives in the universe Ω, which happens to be a physics simulation. In Ω, time runs discretely by seconds. Position, velocity, and acceleration also exist in Ω, and are defined to be $v_\Omega(t) = y_\Omega(t+1) - y_\Omega(t)$ and $a_\Omega(t) = v_\Omega(t+1) - v_\Omega(t)$ for some integer t. In Ω, all objects receive a uniform gravitational acceleration $a_\Omega(t) = -g$, like that on Earth.

Suppose that at $t = 0$ and $y_\Omega(0) = 0$, Bob throws a ball upward, with initial non-zero velocity v_0. A ball with identical initial conditions is thrown on Earth.

Let t_Ω be the maximal integer such that $y_\Omega(t_\Omega) \geq 0.$, and t_E the maximal real number such that $y_E(t_E) \geq 0$ Find the best relation between t_Ω and t_E.

(A) $t_\Omega < t_E$

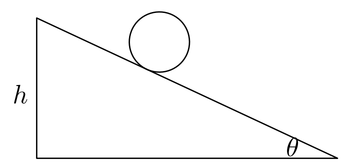
(B) $t_\Omega \leq t_E$

(C) $t_\Omega = t_E$

(D) $t_\Omega \geq t_E$

(E) $t_\Omega > t_E$

22. A solid ball with rotational inertia $\frac{2}{5}mr^2$ is released from rest down inclines of various inclination angles θ but all through a fixed vertical height h. At what angle will the ball stop rolling without slipping assuming the coefficient of static friction μ is equal to the coefficient of kinetic friction?



(A) $\arctan(\mu)$

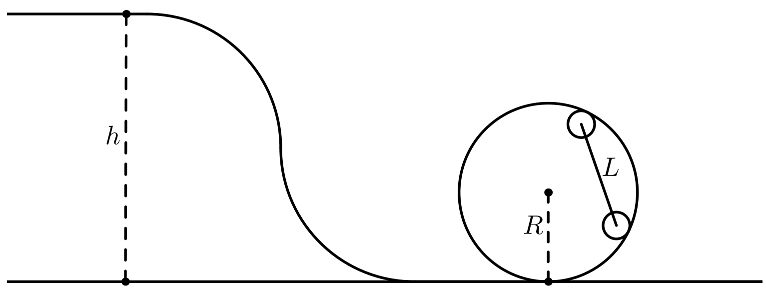
(B) $\arctan(\frac{7}{2}\mu)$

(C) $\arctan(\frac{3}{10}\mu)$

(D) $\arcsin(2\mu)$

(E) $\arccos(\frac{4}{3}\mu)$

23. Two small, equal masses are connected with a rod of length L. They are given a small nudge off a cliff with height h, and they slide through a circular loop of radius $R = \frac{L}{\sqrt{2}}$, as shown in the figure. Given that the masses always remain in contact with the track, find the minimum height h.



(A) $1 + \frac{\sqrt{30}}{4}$

(B) $1 + \frac{\sqrt{32}}{4}$

(C) $1 + \frac{\sqrt{34}}{4}$

(D) $1 + \frac{\sqrt{36}}{4}$

(E) $1 + \frac{\sqrt{38}}{4}$

24. Barkimedes and Deshawn Williams are working on a lab. They each independently measure the volume of a mass dispeged in a graduated cylinder. They decide to do a weighted average to minimize the error in the experiment. What is the weight in their measurements given that Deshawn's error in measurement is 3 times smaller that of Barkimedes error in measurement?

(A) Deshawn's measurement is weighted 3 times as much to result in minimum error

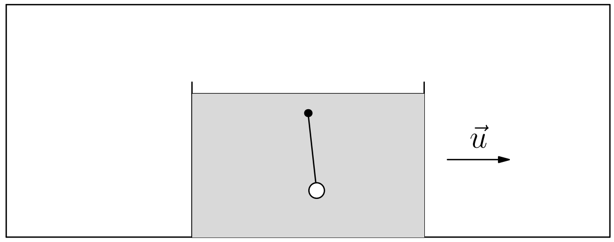
(B) Deshawn's measurement is weighted 4 times as much to result in minimum error

(C) Deshawn's measurment is weighted 9 times as much to result in minimum error

(D) Deshawn's measurment is weighted 10 times as much to result in minimum error

(E) The error will not be minimized by a weighted average

25.



A truck is driving with velocity $v = 8m/s$ around a circle of radius r. Inside the truck, a jar of water is sliding with $u = 3.2m/s$ with respect to the truck in a direction perpendicular to the truck's traveling direction and gravity. Within the water of density $\rho_w = 1000kg/m^3$, a sphere with density $\rho_s = 21000kg/m^3$ is attached to a string of length l to form a simple pendulum. (See picture for cross-section of truck; truck is traveling into the page, and turning towards the left)

Find the radius r such that the small angle oscillation frequency of the pendulum is the same as that of an identical pendulum in "normal conditions" (outside the jar and truck).

(A) 20.0 m

(B) 21.5 m

(C) 25.6 m

(D) 26.9 m

(E) Impossible. No such r allows the frequency to be the same.