

CVMS CONTEST

Mock AMC 12

Credits to jeffisepic, budu, and welpwelphahaha

Also a huge thanks to our testsolvers, flyrain, Anno, Math1331Math, sas4, and sulii!

Σ RULES AND INFO

1. HONESTY!!!! Please do not cheat or disobey our rules. Remember, this is to learn, winning by cheating does not feel like a victory at all.
2. Time limit: 105 minutes
3. No excess material other than paper and writing instruments. If you really want a ruler or compass or protractor, fine. Keep in mind, you can solve all of these problems without these instruments. NO CALCULATORS ARE ALLOWED.
4. Scoring: 6 points for each correct answer, 1.5 points for each problem left blank, 0 for each incorrect answer.
5. Have fun and hopefully you will enjoy this contest hosted by CVMS!

If you find any errors in the problems, PM welpwelphahaha, jeffisepic, and budu on AoPS.

1. What is the probability that in 10 coin flips, at least 6 will be heads?

- (A) $\frac{193}{512}$ (B) $\frac{99}{256}$ (C) $\frac{105}{256}$ (D) $\frac{1}{2}$ (E) $\frac{151}{256}$

2. Jeffisepic takes a random number, x , divides it by 2, squares it, and adds 3. If his answer is less than 10, how many different integers can the number x be?

- (A) 5 (B) 6 (C) 7 (D) 10 (E) 11

3. In a lottery, n people enter, with an admission fee. The lottery leader gives y percent of the total amount of money he received to the winner. Everyone is expected to lose 3 dollars every time they enter this! How much does the admission fee cost, in terms of y and n ?

- (A) $\frac{300}{100-y}$ (B) $\frac{300}{y}$ (C) $\frac{300n}{100-y}$ (D) $\frac{300n}{y}$ (E) $\frac{300}{ny}$

4. Every permutation of jeffisepic is listed out. What is the probability that if one permutation is chosen randomly, that the arrangement "pie" is in the permutation, in that order?

- (A) $\frac{1}{45}$ (B) $\frac{2}{45}$ (C) $\frac{4}{45}$ (D) $\frac{1}{15}$ (E) $\frac{2}{15}$

5. A circle of radius 6 is divided into two sectors, the smaller one with a central angle of 60° . If the two sectors are rolled into cones, what is the volume of the larger cone?

- (A) $\frac{5\sqrt{11}\pi}{3}$ (B) 25π (C) $\frac{25\sqrt{3}\pi}{5}$ (D) $\frac{25\sqrt{11}\pi}{3}$ (E) $\frac{6\sqrt{3}\pi}{5}$

6. A calculator has 4 different buttons that add, subtract, multiply, and divide 2. There is also a special installation so the calculator can square numbers. What is the least number of operations to reach 2016 if the calculator starts at 1?

- (A) 8 (B) 9 (C) 10 (D) 13 (E) 21

7. There are 24 candies and 4 children. How many ways can the children divide the candy if each child must get at least 3, the first child cannot get more than 6 and the last child cannot get less than 6?

- (A) 105 (B) 144 (C) 156 (D) 164 (E) 210

8. Compute

$$10 * \binom{3}{0} + 9 * \binom{4}{1} + 8 * \binom{5}{2} + \dots + \binom{12}{9}.$$

- (A) 1001 (B) 1287 (C) 1716 (D) 2002 (E) 2016

9. A number n is inputted into the factor factory. The number of factors in n is computed. First, budu plugs a number in. Then, budu wants to plug in the result of the previous attempt forever until he gets the number 2. The number of times he plugs a number in is the score he gets. For example, if budu plugs 16 in, he gets 16, 5, 2, so his score is 2. What is the product of all non-zero digits of the least integer that he could plug in so that he gets a score greater than 5?

- (A) 6 (B) 8 (C) 12 (D) 14 (E) 20

10. Determine the sum of all 3 digit numbers such that when multiplied by 325, the result ends in 325.

- (A) 3454 (B) 5951 (C) 6492 (D) 9782 (E) 11902

11. A square is inscribed in a quarter circle with radius 2, so that 2 vertices are on the arc of the circle. What is the area of the square?

- (A) $\frac{3}{2}$ (B) $\frac{8}{5}$ (C) $\frac{5}{3}$ (D) 2 (E) $\frac{5}{2}$

12. Budu challenges welpwelpahahaha to a match. Each turn, the player has a 30% chance of winning. The order of turns are budu, welpwelpahahaha, welpwelpahahaha, budu, continues until someone wins. What is the probability that budu will win?

- (A) $\frac{37}{91}$ (B) $\frac{65}{126}$ (C) $\frac{3}{7}$ (D) $\frac{79}{149}$ (E) $\frac{1}{2}$

13. The roots of the polynomial $P(x) = x^5 - 2x^4 - 2x^3 + 4x^2 - \frac{1}{2}$ are $r_1, r_2, r_3, r_4,$ and r_5 . Compute $(1 - r_1^2)(1 - r_2^2)(1 - r_3^2)(1 - r_4^2)(1 - r_5^2)$.

- (A) $-\frac{5}{4}$ (B) $-\frac{3}{2}$ (C) $-\frac{15}{4}$ (D) $\frac{9}{4}$ (E) $\frac{2}{3}$

14. How many positive integers less than 1000 have three times as many composite factors as prime factors?

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 11

15. All the positive integers from 1 to 256 are written in base 4. Find the sum, in base 10, of all the numbers whose digits in base 4 sum up to 10.

- (A) 1636 (B) 1922 (C) 2085 (D) 2125 (E) 2340

16. In $\triangle ABC$, $\angle ACB = 15^\circ$ and $\angle CAB = 135^\circ$. Extend \overline{AB} past B to point D such that $\angle ACD = 30^\circ$. What is the ratio of the area of $\triangle BCD$ to the area of $\triangle ABC$?

- (A) $\sqrt{3} - 1$ (B) $\sqrt{3}$ (C) $\frac{3\sqrt{2}}{2}$ (D) $2\sqrt{2}$ (E) $\sqrt{3} + 1$

17. What is the probability that any given palindrome less than 10000 is a multiple of 7?

- (A) $\frac{1}{9}$ (B) $\frac{7}{66}$ (C) $\frac{31}{198}$ (D) $\frac{16}{99}$ (E) $\frac{17}{99}$

18. Budu, Jeffisepic, Welpwelpahaha, and Jeffisepic's dad walk, run, ride a bicycle, and drive around a circular track at a constant rate of 6, 8, 15 and 56 miles per hour respectively. Assuming no one gets knocked over, how many times will 2 or 3 of them meet one another before all 4 of them meet besides at the start?

- (A) 61 (B) 65 (C) 75 (D) 79 (E) 90

19. Let a perfect cylinder be a cylinder with diameter and height the same. A perfect cylinder is inscribed in a sphere with radius 3. Then a perfect cylinder that is inscribed in a sphere is inscribed in the previous cylinder. This process occurs infinitely times. What is the sum of the volume of all the spheres?

- (A) $\frac{288\pi+72\sqrt{3}\pi}{7}$ (B) $144\sqrt{3}\pi$ (C) $\frac{36\pi+72\sqrt{3}\pi}{5}$ (D) $144\pi + 36\sqrt{3}$ (E) $18 + 72\sqrt{3}$

20. How many ordered pairs of positive integers (a, b) are there such that $a, b \leq 50$ and the quadratic $x^2 + ax + b$ has at least 1 integer root?

- (A) 73 (B) 79 (C) 84 (D) 96 (E) 106

21. The point $(13, 13)$ is reflected across the x axis, the y axis, the lines $y = -\frac{3}{2}x$ and $y = -\frac{2}{3}x$. After connecting the five points, what is the area of the resulting pentagon?

- (A) 676 (B) 690 (C) 770 (D) 816 (E) 980

22. Welpwelpahaha finds the number of perfect squares in the form WE1WE, a five-digit number. Jeffisepic finds the number of perfect squares in the form of FATCAT, a six-digit number, such that the value of C is exactly 1 greater than the value of F. Repeating letters are the same. What is the positive difference between Jeffisepic's count and Welpwelpahaha's count?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

23. Find the last 3 digits of $8^{2015^{2016}}$.

- (A) 288 (B) 328 (C) 568 (D) 728 (E) 968

24. A bulldozer in the form of a unit square is rotated 75° around its center. What is the area the bulldozer sweeps away?

- (A) $\pi + \sqrt{2}$ (B) $\frac{\pi+1}{2} + \sqrt{6}$ (C) $\frac{5\pi}{12} - \frac{1}{2} + \frac{\sqrt{3}}{6}$ (D) π (E) $\frac{\pi}{4} + 2 - \sqrt{2}$

25. A computer error while grading the MOCK AMC 11 accidentally makes correct answers to even-numbered problems worth 7 instead of 6 points. What is the sum of all possible scores in this MOCK AMC 11 (scoring guidelines on 1st page) with the computer error?

- (A) 24048.5 (B) 24246.5 (C) 28750 (D) 30268.5 (E) 31736