

Intellectuality123's Summer Math Contest!

Hello everyone! Welcome to Intellectuality123's Summer Math Competition! Here are the general rules:

This test will consist of 20 questions and you will have 30 minutes to complete this test.

When you are done, please PM your answers to Intellectuality123.

If you notice a typo or wrong wording, please PM me immediately.

Note: I decided to spread the difficulty of this test throughout. You may do a really easy problem, then encounter a really hard problem.

Good luck and have fun!

1. What is the sum of all 5-digit palindromes less than 12530 but greater than 11000?

2. What is the shortest distance between the circles

$$(x - 3)^2 + (y + 2)^2 = 9 \text{ and } (x - 8)^2 + (y + 14)^2 = 16$$

3. Calculate the resulting temperature when 30 grams of 90-degree water is mixed with 70 grams of 70-degree water.

4. What is the next term(x) in the following sequence?

$$1, 3, 10, 28, 63, 121, 208, x$$

5. How many arrangements of the word MATHEMATICS include the word MATH,

given that the 2 Ms, Ts, and As are indistinguishable?

6. The 3 hands on a clock overlap at 12:00. After x minutes, the hand that displays seconds

bisects the acute angle formed by the minute and hour hands for the first time.

Find x . Express your answer as a common fraction.

7. The side length of a square ABCD is 2 units. Point M is on CD, and AM bisects angle CAD. What is the length of AM?

Express your answer in simplest radical form.

8. A coach must choose 5 starters from a team of 12 players. Adam and John always play at the same time.

How many different ways can the coach choose the starters?

9. How many positive integers not exceeding 2001 are multiples of 3 and/or 4, but not 5?

10. Find the area of a triangle with side lengths 51, 52, and 53.

11. Which is larger, 2^{3000} or 3^{2000} ?

12. Which is larger, $\sqrt[3]{5}$ or $\sqrt{3}$?

13. If the $n=602975024681576789349125$, find the value of $\lfloor \frac{n}{10^{\lfloor \log(n) \rfloor}} \rfloor$, where $\log(n)$ is the common logarithm to the base 10? Note that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to n .

14. How many integers less than 468 but greater than 0 are relatively prime to 468?
15. Given that f is a quadratic polynomial satisfying $f(1) = 3, f(2) = 6, f(3) = 10$, find $f(4)$.
16. What is the area of the figure enclosed by the lines $y = 3x + 2, x = 5, y = -2x - 8$?
17. There are 12 light switches in a row that are on. 4 of the switches will be turned off with the following conditions:
- 1.No adjacent switches will be turned off.
 2. No switches in each end will be turned off.

How many ways are there to turn off the switches?

18. Joe assigns different values 0 through 9 to the different unknown digits $A, B, C, D, E, F, G, H, I$, and J such that the number $ABCD$ (A is thousands place, B is hundreds, etc.) plus the number $EFGHI$ (E is ten-thousands place, etc.) equals the number $J2013$.(J is ten-thousands place, 2 is thousands place, etc.) Find the value of $A + B + C + D + E + F + G + H + I$.

19. We define an operation $a \otimes b = (a^{th} \text{ prime number})(b^{th} \text{ composite number})$. Find the absolute difference between $25 \otimes 36$ and $36 \otimes 25$.

20. Given

$$a + b = 15$$

$$c - a = a + 2$$

$$d - b = a$$

$$a + b + c + d = 42$$

find the value of

$$2a^2 + 3b - 7c + 8d.$$

And we're done!

Credits given to...

Intellectuality123 (test-creator)

dhwang314, ghghghghghghghgh, and Tega (test-solvers)