Let $v_p(n)$ be the largest power of a prime p that divides n. Then the answer is $\max[v_2(n), v_5(n)]$.

First we only consider those n for which $\gcd(n, 10) = 1$. We will show that $\frac{1}{n}$ has no non-repeating part.

First we understand what we are doing when we divide 1 by n. Since $n > 1$, the integer part of the quotient will be 0. After we put the decimal point, the new dividend becomes 10, and the first digit of the fractional part of the quotient is n_1, where nn_1 is the largest multiple of n less than 10. Now the dividend is $(10 - nn_1) \cdot 10$, and the digit after n_1 is n_2, where nn_2 is the largest multiple of n less than $(10 - nn_1) \cdot 10$, and we continue like this, with $n_3, n_4, \dots$ as the succeeding digits in the quotient. Note that $n_1, n_2, \dots \in \{0, 1, \dots, 9\}$. We write this down as a sequence of steps as follows:

$$\begin{aligned} &(10, n_1 n) \\ \mapsto\ &(10^2 - 10n_1 n, n_2 n) \\ \mapsto\ &(10^3 - 10^2 n_1 n - 10n_2 n, n_3 n) \\ &\vdots \end{aligned}$$

Note that in every step, we choose the n_i in such a way, so that the first number of the pair, is as small as possible for a fixed n.

Lemma: There exists an x such that the remainder we get after the xth step is 1.

Proof:Choose x such that the smallest repunit (numbers of the form $111\dots1$) with x digits is divisible by n. Call this repunit r_x. We can always do so as long as $\gcd(n, 10) = 1$. We let our sequence go on for x steps, and the xth pair is

$$(10^x - n(10^{x-1}n_1 + 10^{x-2}n_2 + \dots + 10n_{x-1}), n_x n).$$

Then the remainder after the xth step would be $10^x - n(10^{x-1}n_1 + 10^{x-2}n_2 + \dots + 10n_{x-1} + n_x)$. So in order for this to be 1, we need $n(10^{x-1}n_1 + 10^{x-2}n_2 + \dots + 10n_{x-1} + n_x) = 10^x - 1$ which is nothing but $9r_x$. We show that this is the case. Let $n(10^{x-1}n_1 + 10^{x-2}n_2 + \dots + 10n_{x-1} + n_x) = s$. Then we wish to show that it is possible to have $\frac{s}{n} = 9\frac{r_x}{n}$. Then we could argue that $\frac{s}{n}$ must equal $9\frac{r_x}{n}$, due to our as-large-as-possible criterion. Note that $10^x - s$ is positive and as small as possible. This means that s is as large as possible. We also have $s < 10^x$, which means s must have x digits. The largest number with x digits is $9r_x$. Note that $r_x < 10^x$, so $\frac{r_x}{n} < 10^{x-1}$, and $9\frac{r_x}{n} < 10 \cdot 10^{x-1} = 10^x$. So the number of digits of $9\frac{r_x}{n}$ is less than or equal to x. Since $10^{x-1}n_1 + 10^{x-2}n_2 + \dots + 10n_{x-1} + n_x = \frac{s}{n}$ is an x digit number, the value of $10^{x-1}n_1 + 10^{x-2}n_2 + \dots + 10n_{x-1}$ can be made $9\frac{r_x}{n}$, and thus we must have $s = n(10^{x-1}n_1 + 10^{x-2}n_2 + \dots + 10n_{x-1}) = 9r_x = 10^x - 1$. So the remainder after the xth step is $10^x - (10^x - 1) = 1$. $\square$

Now the $(x+1)$th step is

$$((10^x - (10^x - 1)) \cdot 10, n_{x+1}n) = (10 \cdot n_1 n).$$

This means that the digits of the quotient again starts repeating from n_1, and as such the length of the non-repeating part is 0.

Now suppose $n = 2^a 5^b k$ where $a, b \geq 0$ and $\gcd(k, 10) = 1$. Then $\frac{1}{n} = \frac{1}{2^a 5^b} \cdot \frac{1}{k}$. Since the non-repeating part of $\frac{1}{k}$ is 0, the non-repeating part of $\frac{1}{n}$ is simply $\max(a, b)$.

Thus we have our answer as $\max[v_2(n), v_5(n)]$.